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Soft Gluon Resummation for Polarized Deep-inelastic Production of Heavy Quarks

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Abstract

We present the threshold resummation for the cross section of heavy quark production in polarized deep-inelastic scattering to next-to-leading logarithmic accuracy and in single-particle inclusive kinematics. We expand our resummed result in α_s to next-to-leading and next-to-next-to-leading order and study the impact of these higher order corrections on the charm structure function g_1^c in the kinematical range accessible to the HERMES and COMPASS experiments.

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1 Introduction

Spin dependent deep-inelastic lepton-hadron scattering has received much interest during the past years, see Refs.[1, 2, 3, 4] for reviews. Presently, next-to-leading order analyses of the experimental data for the structure function g_1 allow for the extraction of polarized parton distributions. However, these are considerably less constrained than their unpolarized counterparts. This is particularly true for the gluon spin distribution $\Delta\phi_{g/P}$, which is currently determined from scaling violations. Numerous polarized hard scattering processes are sensitive to the gluon spin distribution, and the first moment of $\Delta\phi_{g/P}$ contributes to the sum rule for the nucleon's spin content [5, 6].

Deep-inelastic leptoproduction of heavy quarks proceeds predominantly through the boson-gluon fusion process, and as such yields important information about the gluon distribution inside the proton. Polarized leptoproduction of charm quarks is therefore promising for an experimental determination of $\Delta\phi_{g/P}$. Presently, this measurement can be realized by the HERMES experiment [7]. The upcoming COMPASS experiment [8] is dedicated to measure $\Delta\phi_{g/P}$ directly through the boson-gluon fusion process and subsequent polarized open charm production. COMPASS will probe both the region of photoproduction and deep-inelastic scattering and extend the kinematical range accessible to the HERMES experiment.

In general, fixed target experiments, such as HERMES or COMPASS with centre-of-mass energies of $\sqrt{S} = 7.4\text{GeV}$ and 14.1GeV respectively, operate rather close to the threshold for charm pair-production. In this region of phase space, the perturbatively calculable hard scattering cross section has potentially large higher order double-logarithmic threshold corrections. Thus, a resummation of these Sudakov logarithms to all orders in perturbation theory is required in order to regain control over the perturbative expansion.

The purpose of this letter is to carry out this Sudakov resummation, which is done in section 2. We work to next-to-leading logarithmic (NLL) accuracy (see Ref.[9] for a recent review), and our final result provides an approximate sum of the complete perturbative expansion if, at each order, the Sudakov corrections dominate. Previously, studies of threshold resummation for polarized scattering processes have been performed for the polarized Drell-Yan cross section in Ref.[10]. By re-expanding our resummed cross section in α_s to next-to-leading (NLO) and next-to-next-to-leading order (NNLO), we provide in section 3 estimates of the size of the exact higher order corrections[1], the calculation of which has not yet been completed [11, 12]. Our estimates are valid in the region dominated by threshold logarithms and we study their impact on the charm structure function g_1^{c}. In the kinematical region relevant to HERMES and COMPASS, we investigate the sensitivity to the gluon spin distribution $\Delta\phi_{g/P}$ and the dependence on the factorization scale. Section 4 contains our conclusions.

2 Resummed differential cross section

We begin with the definition of the exact single-particle inclusive (1PI) kinematics. We study the reaction of lepton (l) – proton (P) scattering,

$$\mathrm{l}(l) + \mathrm{P}(p) \to \mathrm{l}(l-q) + \mathrm{Q}(p_1) + X[\overline{\mathrm{Q}}](p_2')\,, \tag{1}$$

where Q is a heavy quark and $X[\overline{\mathrm{Q}}]$ denotes any allowed hadronic final state containing at least the heavy antiquark. Neglecting electroweak radiative corrections[2], for longitudinally polarized

[1] For the exact NLO corrections to polarized heavy quark photoproduction cf. Refs.[13, 14, 15, 16].

[2] QED radiative corrections have been studied in the leading-logarithmic approximation in Ref.[17].

protons, the difference of cross sections for reaction (1) may after summation over X be written as [11, 12, 18, 19, 20]

$$\frac{d^5\sigma^{\leftrightarrows}}{dx\,dy\,d\phi\,dT_1\,dU_1} - \frac{d^5\sigma^{\rightrightarrows}}{dx\,dy\,d\phi\,dT_1\,dU_1} = \tag{2}$$
$$\frac{4\alpha^2}{Q^2}\left[\left(2-y-\frac{2M^2x^2y^2}{Q^2}\right)\frac{d^2g_1(x,Q^2,m^2,T_1,U_1)}{dT_1\,dU_1} - \frac{4M^2x^2y^2}{Q^2}\frac{d^2g_2(x,Q^2,m^2,T_1,U_1)}{dT_1\,dU_1}\right],$$

where α and M are the fine structure constant and the proton mass, and ϕ denotes the angle between the proton spin and the momentum of the scattered lepton, $(l-q)$ in Eq.(1). The lower arrow on $d^5\sigma$ indicates the polarization of the incoming lepton in the direction of its momentum l, whereas the upper arrow on $d^5\sigma$ represents the polarization of the proton, which is parallel or anti-parallel to the polarization of the incoming lepton.

The functions $d^2g_k/dT_1\,dU_1$, $k=1,2$ are the double-differential deep-inelastic polarized heavy quark structure functions. The kinematic variables Q^2, x, y are defined by

$$Q^2 = -q^2 > 0 \quad , \quad x = \frac{Q^2}{2p\cdot q} \quad , \quad y = \frac{p\cdot q}{p\cdot l}\,. \tag{3}$$

Additionally, we also have the overall invariants

$$S = (p+q)^2 \equiv S' - Q^2, \quad T_1 = (p-p_1)^2 - m^2, \quad U_1 = (q-p_1)^2 - m^2, \quad S_4 = S + T_1 + U_1 + Q^2. \tag{4}$$

Only the structure function g_1 is relevant asymptotically for $M/Q \ll 1$. It contains leading twist two operators and has a parton model interpretation. We therefore restrict ourselves to g_1 only. Furthermore, we neglect the contributions from initial state quarks and antiquarks. This is justified as they vanish at leading order. We consider only the gluon-initiated partonic subprocess

$$\gamma^*(q) + g(k) \longrightarrow \mathrm{Q}(p_1) + X'[\overline{\mathrm{Q}}](p_2')\,, \tag{5}$$

where $k = z\,p$. The partonic invariants are

$$s = (k+q)^2 \equiv s' + q^2, \quad t_1 = (k-p_1)^2 - m^2, \quad u_1 = (q-p_1)^2 - m^2, \quad s_4 = (p_2')^2 - m^2. \tag{6}$$

The invariant s_4 parametrizes the inelasticity of the partonic reaction (5) in 1PI kinematics. This distance from threshold is conveniently measured in terms of a dimensionless weight function w_S [21],

$$w_S = \frac{s_4}{m^2} \simeq \frac{2\bar{p}_2\cdot k_S}{m^2} \equiv \frac{2\zeta\cdot k_S}{m}\,. \tag{7}$$

The 1PI kinematics vector ζ is defined by splitting the recoil momentum $p_2' = \bar{p}_2 + k_S$ into the momentum $\bar{p}_2$ at threshold, and the momentum k_S of any additional (soft) radiation above threshold.

In double-differential form, $d^2g_1/dT_1\,dU_1$ factorizes as

$$S'^2\frac{d^2g_1(x,Q^2,m^2,T_1,U_1)}{dT_1\,dU_1} = \int\limits_{z^-}^{1}\frac{dz}{z}\,\Delta\phi_{g/P}(z,\mu^2)\,\Delta\omega_g\Big(\frac{x}{z},s_4,t_1,u_1,Q^2,m^2,\mu^2,\alpha_s(\mu)\Big)\,. \tag{8}$$

The polarized gluon distribution in the proton is denoted by $\Delta\phi_{g/P}$ and z is the momentum fraction with $z^- = -U_1/(S'+T_1)$. The dimensionless function $\Delta\omega_g$ describes the hard scattering

process and depends on the partonic invariants t_1, u_1 and s_4, defined in Eq.(6). The factorization scale is denoted by μ and throughout this paper taken equal to the renormalization scale.

Sufficiently close to threshold, g_1 in Eq.(8) is dominated by those higher order contributions that contain plus-distributions of the type

$$\left[\frac{\ln^l(s_4/m^2)}{s_4}\right]_+ = \lim_{\Delta\to 0}\left\{\frac{\ln^l(s_4/m^2)}{s_4}\theta(s_4-\Delta) + \frac{1}{l+1}\ln^{l+1}\left(\frac{\Delta}{m^2}\right)\delta(s_4)\right\}. \tag{9}$$

They result from imperfect cancellations between soft and virtual contributions to the cross section. At order $\mathcal{O}(\alpha_s^{i+2})$, $i = 0, 1, \ldots$ we refer to the corrections with $l = 2i + 1$ as leading logarithmic (LL), to the ones with $l = 2i$ as next-to-leading logarithmic (NLL), etc.

The Sudakov resummation of these singular functions in Eq.(9) uses the methods and results of Refs.[21, 22, 23, 24]. It rests upon the factorization of $\Delta\omega_g$ into separate functions $\Delta\psi_{g/g}$, S, and H_g for the jetlike-, soft, and off-shell quanta. This factorization, valid in the threshold region of phase space, implies a decomposition [24] of the total weight function w,

$$w = w_\psi + w_S = w_1\left(\frac{-u_1}{m^2}\right) + w_S\,, \tag{10}$$

which expresses the overall weight in terms of the individual weights w_ψ and w_S. Each of the functions $\Delta\psi_{g/g}(w_1)$, $S(w_S)$, and H_g, which are conveniently computed in $\zeta\cdot A = 0$ gauge, organizes contributions corresponding to a particular set of quanta and thereby only depends on its own individual weight function.

It is convenient to consider these functions in moment space, defined by the Laplace transform with respect to the overall weight w,

$$\tilde{f}(N) = \int_0^\infty dw\, e^{-Nw} f(w)\,. \tag{11}$$

Under the assumption of factorization the partonic cross section $\Delta\omega_g$ in moment space can then be written in a factorized form, up to corrections of order $1/N$, as [21, 23, 24, 25]

$$\begin{aligned}&\Delta\tilde{\omega}_g(N, t_1, u_1, Q^2, m^2, \mu^2, \alpha_s(\mu)) = \\ &\quad H_g(\zeta, Q^2, m^2, \alpha_s(\mu))\left[\frac{\Delta\tilde{\psi}_{g/g}(N_u, k\cdot\zeta/\mu)}{\Delta\tilde{\phi}_{g/g}(N_u,\mu)}\right]\tilde{S}\left(\frac{m}{N\mu},\zeta\right).\end{aligned} \tag{12}$$

where $N_u = N(-u_1/m^2)$. The N-dependence in each of the functions of Eq.(12) exponentiates.

The jet and soft functions, $\Delta\psi_{g/g}$ and S in Eq. (12), can each be represented as operator matrix elements [22, 23, 24, 25]. Dependence on the spin degrees of freedom can be kept explicit in these elements, such that the methods of Refs.[21, 22, 23, 24] for the resummation of the singular functions in Eq.(9) via appropriate evolution equations can be generalized in a straightforward way to account for initial state polarizations. The jet-function $\Delta\tilde{\psi}_{g/g}$ obeys two evolution equations, namely the renormalization group equation and an equation describing the energy dependence via gauge-dependence [21, 22, 26]. Solving both equations sums Sudakov double logarithms. Mass factorization introduces the density $\Delta\tilde{\phi}_{g/g}$ as a counterterm and requires the choice of a scheme. In the $\overline{\text{MS}}$-scheme $\Delta\tilde{\phi}_{g/g}$ has no double logarithms. The soft function S obeys a renormalization group equation, which resums all NLL logarithms originating from soft gluons not already accounted for in $\Delta\tilde{\psi}_{g/g}$. In general, the soft and

the hard function S and H_g depend also on the colour structure of the underlying scattering reaction, but for deep-inelastic production of heavy quarks this dependence is trivial [25].

The final result for the hard scattering function $\Delta\tilde{\omega}_g$ in the $\overline{\rm MS}$-scheme in moment space resums all $\ln N$ in single-particle inclusive kinematics. We obtain

$$
\begin{aligned}
\Delta\tilde{\omega}_g^{\rm res}(N, t_1, u_1, Q^2, m^2, \mu^2, \alpha_s(\mu)) \; &= \qquad\qquad\qquad\qquad\qquad\qquad (13) \\
& H_g(\zeta, Q^2, m^2, \alpha_s(m))\; \tilde{S}(1,\zeta) \exp\left\{ 2 \int\limits_\mu^m \frac{d\mu'}{\mu'} \gamma_g\left(\alpha_s(\mu')\right) \right\} \\
& \times\; \exp\left\{ \int\limits_0^\infty \frac{dw}{w} \Big(1 - {\rm e}^{-N_u w}\Big) \Big[\int\limits_{w^2}^1 \frac{d\lambda}{\lambda} A^g(\alpha_s(\sqrt{\lambda} 2k\cdot\zeta)) + \frac{1}{2}\nu_g(\alpha_s(w 2k\cdot\zeta)) \Big] \right\} \\
& \times\; \exp\left\{ \int\limits_m^{m/N} \frac{d\mu'}{\mu'} 2\,{\rm Re}\Gamma_S\left(\alpha_s(\mu')\right) \right\} \exp\left\{ -\, 2 \int\limits_\mu^{2k\cdot\zeta} \frac{d\mu'}{\mu'} \Big(\gamma_g\left(\alpha_s(\mu')\right) - \gamma_{g/g}\Big(N_u, \alpha_s(\mu')\Big) \Big) \right\},
\end{aligned}
$$

where N_u is defined below Eq.(12). To NLL accuracy, as defined below Eq.(9), the product $H_g S$ on the second line of Eq.(13) is determined from matching to the Born result at the scale $\mu = m$. To this accuracy the product $H_g S$ is also insensitive to the choice of treatment of γ_5.

The second exponent in Eq.(13) gives the leading N-dependence of the ratio $\Delta\tilde{\psi}_{g/g}/\Delta\tilde{\phi}_{g/g}$ with

$$
A_g(\alpha_s) \;=\; C_A \frac{\alpha_s}{\pi} + \frac{1}{2} C_A K \left(\frac{\alpha_s}{\pi}\right)^2 + \ldots\,, \qquad\qquad \nu^g(\alpha_s) \;=\; 2 C_A \frac{\alpha_s}{\pi} + \ldots\,, \tag{14}
$$

where $K = C_A(67/18 - \pi^2/6) - 5/9 n_f$ can be inferred from Refs.[27, 28] using Ref.[29] (cf. also Ref.[10]). The scale evolution of the ratio $\Delta\tilde{\psi}_{g/g}/\Delta\tilde{\phi}_{g/g}$ is governed by

$$
\gamma_g(\alpha_s) \;=\; b_2 \frac{\alpha_s}{\pi} + \ldots\,, \tag{15}
$$

$$
\gamma_{g/g}(N, \alpha_s) \;=\; -\frac{\alpha_s}{\pi}(C_A \ln N - b_2) - \left(\frac{\alpha_s}{\pi}\right)^2 \Big(\frac{1}{2} C_A K \ln N\Big) + \ldots\,, \tag{16}
$$

with $\gamma_{g/g}$ calculated in Refs.[27, 28], and the soft anomalous dimension to order α_s is

$$
\Gamma_S(\alpha_s) \;=\; \frac{\alpha_s}{\pi} \left\{ \Big(\frac{C_A}{2} - C_F\Big)(L_\beta + 1) - \frac{C_A}{2} \left(\ln\left(\frac{4(k\cdot\zeta)^2}{m^2}\right) + \ln \frac{m^4}{t_1\, u_1} \right) \right\} + \ldots\,, \tag{17}
$$

with $\beta = \sqrt{1 - 4\, m^2/s}$ and $L_\beta = (1 - 2\, m^2/s)\{\ln(1-\beta)/(1+\beta) + {\rm i}\pi\}/\beta$. We note that the various anomalous dimensions in Eqs.(14)–(17) are identical to NLL accuracy to the corresponding quantities of soft gluon resummation for unpolarized scattering [25].

Eq.(13) represents the central result of this paper. It provides the sum of Sudakov logarithms due to soft gluon emission to all orders in the perturbative expansion and accurate to the next-to-leading logarithm.

3 Finite order results

We expand our resummed result in α_s up to second order so as to provide NLO and NNLO approximations to NLL accuracy for the partonic single-particle inclusive double-differential cross section difference $\Delta\sigma_{\gamma^* g}$.

For the process $\gamma^* + g \longrightarrow \mathrm{Q} + \overline{\mathrm{Q}}$, the single-particle inclusive double-differential cross section difference is related to the hard function $\Delta\omega_g$ of Eq.(8). To NLL accuracy it can be written in a factorized form as

$$
\begin{aligned}
s'^{\,2}\,\frac{d^2\Delta\sigma_{\gamma^* g}(s',t_1,u_1)}{dt_1\,du_1} &= 8\pi^2\,\alpha\,\frac{x}{Q^2}\,\Delta\omega_g\Big(x,s_4,t_1,u_1,Q^2,m^2,\mu^2,\alpha_s(\mu)\Big) \simeq \qquad (18)\\
&\Delta B^{\rm Born}_{\gamma^* g}(s',t_1,u_1)\left\{\delta(s'+t_1+u_1)+\frac{\alpha_s(\mu)}{\pi}K^{(1)}(s',t_1,u_1)+\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 K^{(2)}(s',t_1,u_1)\right\}\,,
\end{aligned}
$$

with the Born level hard part $\Delta B^{\rm Born}_{\gamma^* g}$ given by [30, 31]

$$
\begin{aligned}
\Delta B^{\rm Born}_{\gamma^* g}(s',t_1,u_1) &= \qquad (19)\\
&\alpha\,\alpha_s\,e_{\rm q}^2\,\pi\left[-\frac{t_1}{u_1}-\frac{u_1}{t_1}-2m^2\left(\frac{1}{t_1}+\frac{1}{u_1}+\frac{t_1}{u_1^2}+\frac{u_1}{t_1^2}\right)+2q^2\left(\frac{1}{t_1}+\frac{1}{u_1}+\frac{2}{s'}\right)\right].
\end{aligned}
$$

The NLO and NNLO soft gluons corrections to NLL accuracy are

$$
\begin{aligned}
K^{(1)}(s',t_1,u_1) &= \qquad (20)\\
&2\,C_A\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + \left[\frac{1}{s_4}\right]_+\left\{C_A\left(\ln\left(\frac{t_1}{u_1}\right)+{\rm Re}L_\beta-\ln\left(\frac{\mu^2}{m^2}\right)\right)-2\,C_F\,({\rm Re}L_\beta+1)\right\}\\
&+\delta(s_4)\,C_A\ln\left(\frac{-u_1}{m^2}\right)\ln\left(\frac{\mu^2}{m^2}\right)\,,
\end{aligned}
$$

$$
\begin{aligned}
K^{(2)}(s',t_1,u_1) &= \qquad (21)\\
&2\,C_A^2\left[\frac{\ln^3(s_4/m^2)}{s_4}\right]_+ + \left[\frac{\ln^2(s_4/m^2)}{s_4}\right]_+\left\{3\,C_A^2\left(\ln\left(\frac{t_1}{u_1}\right)+{\rm Re}L_\beta-\ln\left(\frac{\mu^2}{m^2}\right)\right)\right.\\
&\left.-2\,C_A\,(b_2+3\,C_F\,({\rm Re}L_\beta+1))\right\}+\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+\ln\left(\frac{\mu^2}{m^2}\right)\left\{C_A^2\left(-2\ln\left(\frac{t_1}{u_1}\right)\right.\right.\\
&\left.\left.-2{\rm Re}L_\beta+2\ln\left(\frac{-u_1}{m^2}\right)+\ln\left(\frac{\mu^2}{m^2}\right)\right)+2\,C_A\,(b_2+2\,C_F\,({\rm Re}L_\beta+1))\right\}\\
&+\left[\frac{1}{s_4}\right]_+\ln^2\left(\frac{\mu^2}{m^2}\right)\left\{-C_A^2\ln\left(\frac{-u_1}{m^2}\right)-\frac{1}{2}\,C_A\,b_2\right\}\,,
\end{aligned}
$$

with μ the $\overline{\rm MS}$-mass factorization scale, $b_2 = (11C_A - 2n_f)/12$ and L_β given below Eq.(17). We have checked that to NLL accuracy, the result in Eq.(20) agrees with the exact $\mathcal{O}(\alpha_s)$ corrections calculated in Refs.[11, 12].

Next, we perform a numerical investigation of the results (20) and (21). To that end, we relate the total partonic cross section difference to dimensionless coefficient functions $\Delta c_g^{(k,l)}$,

$$
\Delta\sigma_{\gamma^* g}(s',q^2,m^2) = \frac{\alpha\,\alpha_s\,e_{\rm q}^2}{m^2}\sum_{k=0}^{\infty}(4\pi\alpha_s(\mu))^k\sum_{l=0}^{k}\Delta c_g^{(k,l)}(\eta,\xi)\ln^l\frac{\mu^2}{m^2}\,. \qquad (22)
$$

As completely inclusive quantities the functions $\Delta c_g^{(k,l)}$ depend only on the scaling variables

$$
\eta = \frac{s}{4m^2}-1\,, \qquad \xi = \frac{Q^2}{m^2}\,, \qquad (23)
$$

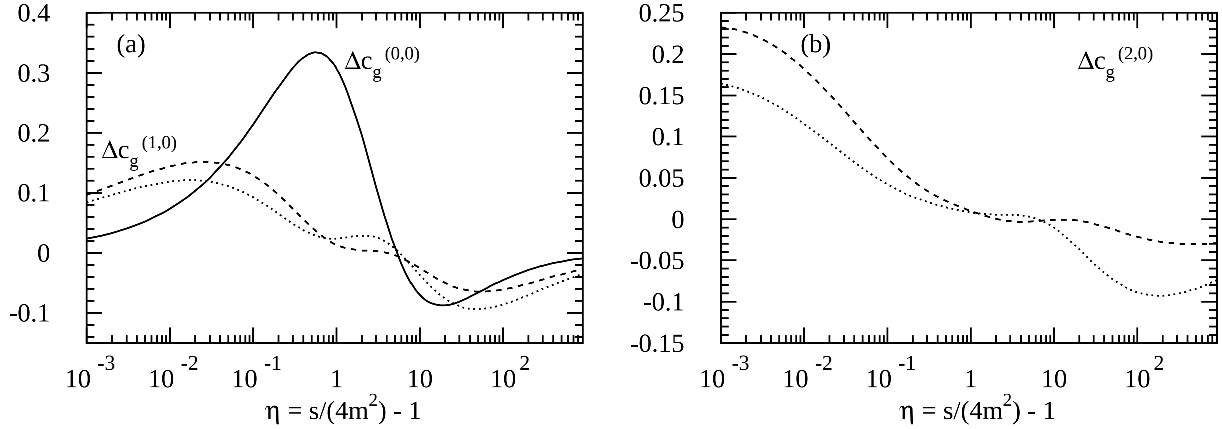

Figure 1: (a) The η-dependence of the coefficient functions $\Delta c_g^{(k,0)}(\eta, \xi)$, $k = 0, 1$ for $Q^2 = 10\,\text{GeV}^2$ with $m = 1.5\,\text{GeV}$. Plotted are the exact result for $\Delta c_g^{(0,0)}$ (solid line) and the LL approximation (dotted line) and NLL approximation to $\Delta c_g^{(1,0)}$ (dashed line). (b) The η-dependence of the coefficient function $\Delta c_g^{(2,0)}(\eta, \xi)$ for $Q^2 = 10\,\text{GeV}^2$ with $m = 1.5\,\text{GeV}$. Plotted are the LL approximation (dotted line) and NLL approximation.

where η is a measure of the distance to the partonic threshold.

In Fig.1 we plot all coefficient functions $\Delta c_g^{(k,0)}; k = 0, 1, 2$, i.e. those not accompanied by scale-dependent logarithms for $Q^2 = 10\,\text{GeV}^2$ and $m = 1.5\,\text{GeV}$. Only the Born function $\Delta c_g^{(0,0)}$ in Fig.1a is known exactly [30, 31]. For $\Delta c_g^{(1,0)}$ in Fig.1a and $\Delta c_g^{(2,0)}$ in Fig.1b we can give estimates to LL and NLL accuracy. In the LL case, we keep only the $[\ln(s_4/m^2)/s_4]_+$ and $[\ln^3(s_4/m^2)/s_4]_+$ terms in Eqs.(20) and (21) respectively. We find both $\Delta c_g^{(1,0)}$ and $\Delta c_g^{(2,0)}$ to be sizable near threshold where the large logarithms dominate, while they tend to be numerically rather small at larger values of η. Moreover, we also investigated some next-to-next-to leading (NNLL) logarithmic contributions[3], such as the Coulomb corrections [11, 12, 20] for $\Delta c_g^{(1,0)}$ and for $\Delta c_g^{(2,0)}$ those NNLL terms, which we obtain from the expansion of the resummed result Eq.(13). Numerically, we found these NNLL terms to have an effect on $\Delta c_g^{(1,0)}$ or $\Delta c_g^{(2,0)}$ of the order of 5% as compared to our NLL corrections.

Since no exact results for $\Delta c_g^{(1,0)}$ and $\Delta c_g^{(2,0)}$ are available we are unable to judge the quality of our approximation. However, similar investigations in the case of unpolarized heavy quark production [25], where exact one-loop results are known [32], suggest that generally NLL accuracy provides a very good approximation of the true result for Q^2 not too large[4], because $K^{(1)}$, $K^{(2)}$ are the same as for the unpolarized case, only $B^{\text{Born}}_{\gamma^* g}$ differs from $\Delta B^{\text{Born}}_{\gamma^* g}$.

Let us now turn to those coefficient functions multiplying scale-dependent logarithms. These are $\Delta c_g^{(1,1)}$ in Fig.2a and $\Delta c_g^{(2,1)}$, $\Delta c_g^{(2,2)}$ in Fig.2b, which we plot for the same values of parameters as in Fig.1. We find in particular the estimates for $\Delta c_g^{(1,1)}$ and $\Delta c_g^{(2,1)}$ to NLL accuracy to be large near threshold. Additionally, we are able to obtain the exact result for the function

[3] For complete resummation to NNLL accuracy one would need to match the resummed result Eq.(13) at NLO, which requires knowledge of all one-loop soft and virtual corrections. These are not yet available [11, 12].

[4] In the regime of large Q^2/m^2 on the other hand, the coefficient functions $\Delta c_g^{(k,0)}$ can be approximated with different methods based on the operator product expansion [20].

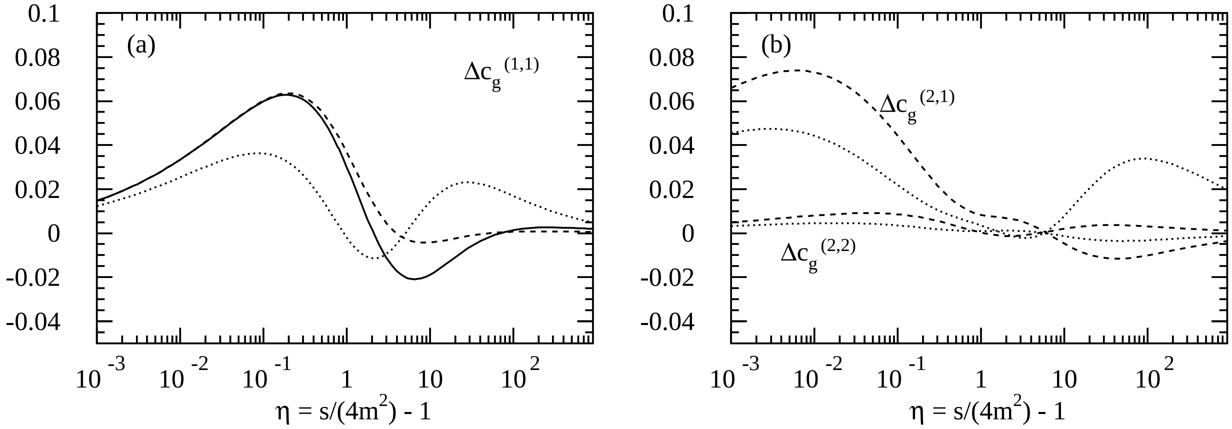

Figure 2: (a) The η-dependence of the coefficient function $\Delta c_g^{(1,1)}(\eta,\xi)$ for $Q^2 = 10\,\mathrm{GeV}^2$ with $m = 1.5\,\mathrm{GeV}$. Plotted are the exact result (solid line), the LL approximation (dotted line) and the NLL approximation (dashed line). (b) The η-dependence of the coefficient functions $\Delta c_g^{(2,l)}(\eta,\xi)$, $l = 1,2$ for $Q^2 = 10\,\mathrm{GeV}^2$ with $m = 1.5\,\mathrm{GeV}$. Plotted are the LL approximation (dotted line) and the NLL approximation (dashed line).

$\Delta c_g^{(1,1)}$ by means of renormalization group methods,

$$\Delta c_g^{(1,1)}\,(\eta(x),\xi) \quad = \quad \frac{1}{4\pi^2}\int\limits_{ax}^{1} dz\ \left(b_2\,\delta(1-z) - \frac{1}{2}\Delta P_{gg}^{(0)}(z)\right)\,\Delta c_g^{(0,0)}\left(\eta\Big(\frac{x}{z}\Big),\xi\right)\,, \tag{24}$$

with $a = 1 + 4m^2/Q^2$ and the one-loop splitting function $\Delta P_{gg}^{(0)}$ given in Refs. [33, 34, 35]. Fig.2a clearly shows that the approximation based on NLL accuracy traces the exact result (24) extremely well even at larger η. For $\Delta c_g^{(2,1)}$ we have checked that NNLL terms obtained from the expansion of the resummed result Eq.(13) have only a small effect.

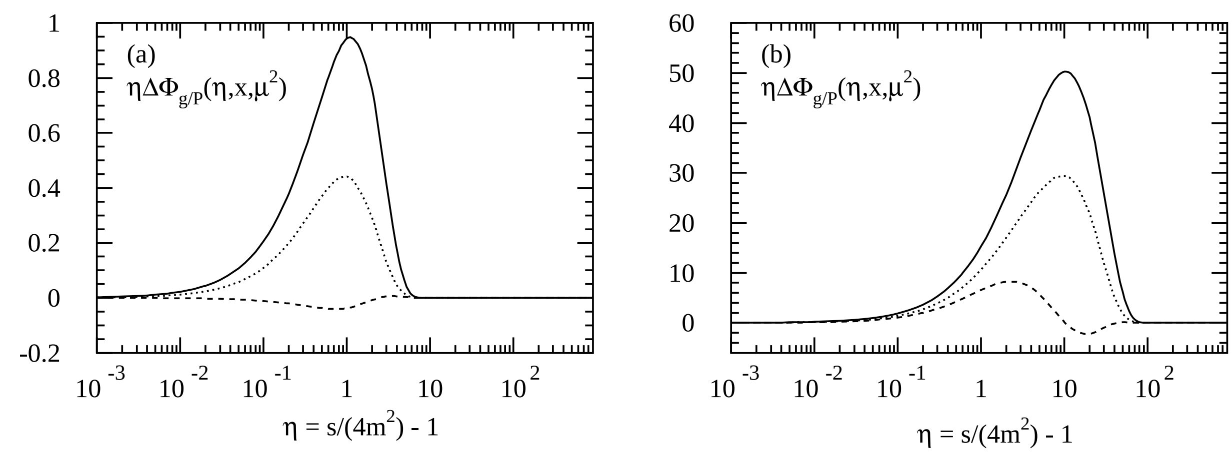

Figure 3: (a) The polarized gluon distribution $\eta\Delta\phi_{g/P}(\eta,x,\mu^2)$ for $x = 0.1$ and $\mu = 1.5\,\mathrm{GeV}$. Plotted are the parametrizations GS A of Ref.[36] (solid), GS C of Ref.[36] (dashed) and GRSV valence of Ref.[37] (dotted). (b) Same as (a) for $x = 0.01$.

Let us now investigate the impact of the approximate higher order perturbative corrections on the inclusive hadronic structure function g_1. In terms of coefficient functions, the charm

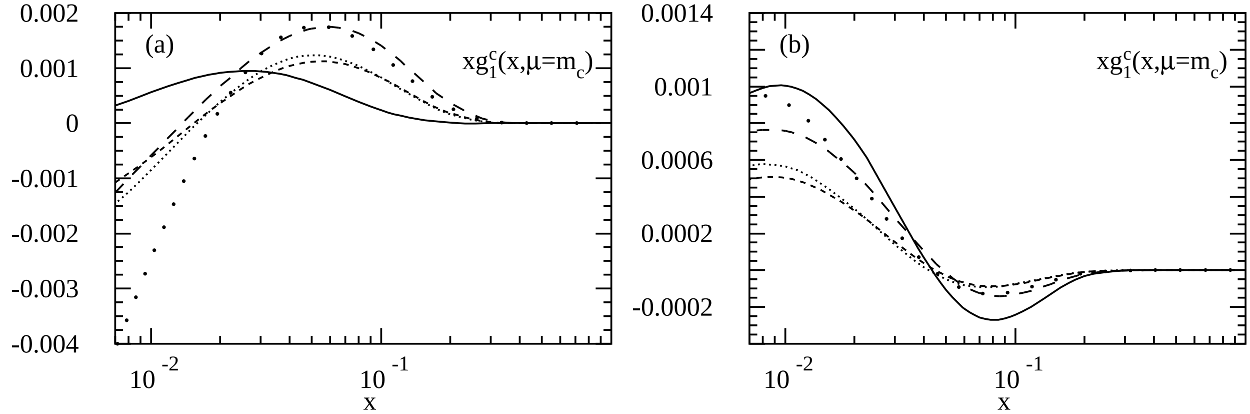

Figure 4: (a) The x-dependence of the charm structure function $xg_1^c(x, Q^2, m^2)$ with the gluon distribution GS A of Ref.[36] for $\mu = m = 1.5\,\mathrm{GeV}$ and $Q^2 = 10\,\mathrm{GeV}^2$. Plotted are the results at leading order (solid), at NLO to LL accuracy (dotted), at NLO to NLL accuracy (dashed), at NNLO to LL accuracy (spaced dotted) and at NNLO to NLL accuracy (spaced dashed). (b) Same as (a) with the gluon distribution GS C of Ref.[36].

structure function g_1^c can be expanded as follows:

$$
\begin{aligned}
&g_1^c(x, Q^2, m^2) \qquad (25)\\
&\quad = \frac{\alpha_s(\mu)\, e_c^2 Q^2}{8\pi^2 m^2 x} \int_{ax}^{1} dz \Delta\phi_{g/P}(z, \mu^2) \sum_{k=0}^{\infty} (4\pi\alpha_s(\mu))^k \sum_{l=0}^{k} \Delta c_g^{(k,l)}(x/z, \xi) \ln^l \frac{\mu^2}{m^2}\\
&\quad = \frac{\alpha_s(\mu)\, e_c^2}{2\pi^2} \int_{-\infty}^{A} d(\log_{10}\eta) \ln 10\ \eta \Delta\phi_{g/P}(\eta, x, \mu^2) \sum_{k=0}^{\infty} (4\pi\alpha_s(\mu))^k \sum_{l=0}^{k} \Delta c_g^{(k,l)}(\eta, \xi) \ln^l \frac{\mu^2}{m^2}\ ,
\end{aligned}
$$

where $a = 1 + 4m^2/Q^2$, $A = \log_{10}(\xi\{1/x - 1\}/4 - 1)$, and $\Delta\phi_{g/P}$ represents the polarized gluon distribution. We recall that the $\overline{\mathrm{MS}}$-scheme has been chosen and that contributions from light initial state quarks are neglected. For $\Delta\phi_{g/P}$ we compare the parametrizations of Refs.[36, 37]. For our analysis at NLO (LO) we use a 2-loop (1-loop) running coupling with $n_f = 3$ light flavours, a charm (pole) mass of $m = 1.5$ GeV [32], and $\Lambda_{\mathrm{QCD}} = 0.232$ GeV.

In Eq.(25), we have chosen to rewrite the standard expression for the convolution of g_1^c in a form, that facilliates the investigation of partonic threshold effects on g_1^c. To do so, we plot $\eta\Delta\phi_{g/P}$ as a function of η in Fig.3, and examine the support it gives to the coefficient functions in the second expression of Eq.(25). By comparing Fig.3 with Figs.1 and 2 one can directly judge over which ranges of η the function $\eta\Delta\phi_{g/P}$ becomes large and hence samples the partonic coefficient functions.

Fig.3 reveals that for all parametrizations this happens indeed in the threshold region for $x \gtrsim 0.01$ and at scales around $Q^2 \simeq 10\,\mathrm{GeV}^2$. We therefore expect our estimates for $\Delta c_g^{(k,l)}$ to provide a good description of the true higher order corrections for g_1^c for $x \gtrsim 0.01$. The gluon densities from set A of Ref.[36] and the valence scenario of Ref.[37] are both positive and similar in shape, while the gluon density from set C of Ref.[36] relaxes the positivity constraint on $\Delta\phi_{g/P}$ and oscillates.[5]

[5] Recent results from the HERMES collaboration [38] indicate a positive ratio of polarized over unpolarized gluon distribution $\Delta\phi_{g/P}/\phi_{g/P}$ at intermediate x.

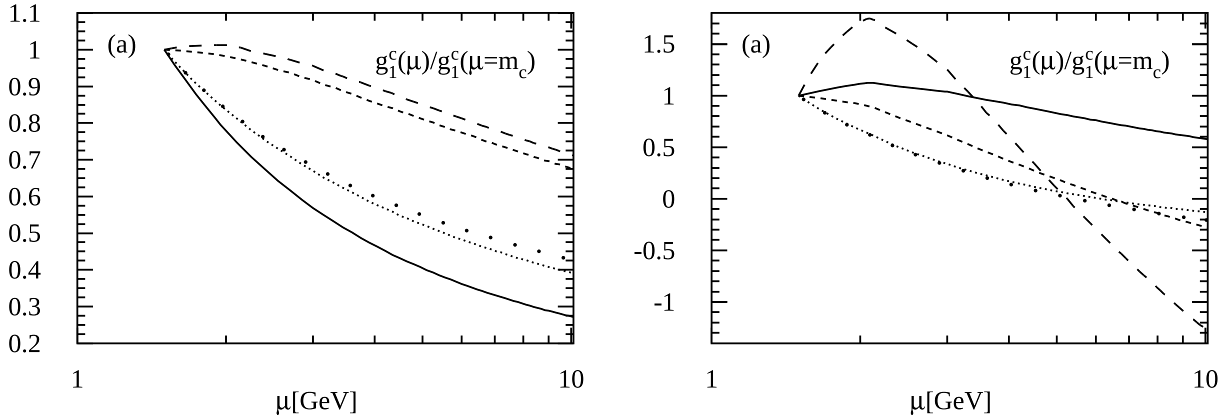

Figure 5: (a) The factorization scale dependence of the charm structure function $g_1^c(x, Q^2, m^2, \mu^2)/g_1^c(x, Q^2, m^2, \mu^2 = m^2)$ with the gluon distribution GS A of Ref.[36] for $m = 1.5\,\mathrm{GeV}$, $Q^2 = 10\,\mathrm{GeV}^2$ and $x = 0.05$. Plotted are the results at leading order (solid), at NLO to LL accuracy (dotted), at NLO to NLL accuracy (dashed), at NNLO to LL accuracy (spaced dotted) and at NNLO to NLL accuracy (spaced dashed). (b) Same as (a) with the gluon distribution GS C of Ref.[36].

Next, we investigate the x-dependence of g_1^c. In particular we are interested in a comparision of our NLO and NNLO estimates (20) and (21) with the leading order result [30, 31, 39]. In Fig.4a we plot the leading order result for xg_1^c as well as the NLO and the NNLO approximations over a range $0.007 \le x \le 1$. We choose $Q^2 = 10\,\mathrm{GeV}^2$, a fixed value of the factorization scale $\mu = m = 1.5\,\mathrm{GeV}$ and the gluon parametrization GS A of Ref.[36]. We obtain similar results with the gluon distribution in the valence scenario of Ref.[37]. We find that the perturbative corrections are sizable, both in the region of intermediate x, around 0.05 and at smaller x, where however, the approximation is less certain to work well. To assess the quality of our approximation, we compare at each order our results to LL and to NLL accuracy. At intermediate x, the small differences between LL and NLL accuracy show a very good stability of the threshold approximation for the description of g_1^c. Towards smaller x however, these deviations increase and indicate that our approximate methods fail for $x \le 0.01$.

In Fig.4b we repeat the plot of xg_1^c for the same parameters as in Fig.4a but with the gluon density GS C of Ref.[36]. Again, the perturbative corrections are sizable over the whole range in x, but the shape of the curves is completely different. Both the coefficient functions and the gluon density oscillate, which leads in particular at intermediate x around 0.1 to destructive interference, with g_1^c being only marginally different from zero.

The final issue we study is the dependence of g_1^c on the mass factorization scale. In general, leading order calculations exhibit a strong sensitivity on the factorization scale, which is usually reduced once higher order corrections are taken into account. In addition, there are general arguments supporting a reduction in scale dependence from including soft gluon effects [40]. Therefore, we are interested in the effect of our NLL-approximate NLO and NNLO results (20) and (21) on the scale dependence of g_1^c in comparision to the leading order calculation [30, 31, 39]. Note however, that the arguments of Ref.[40] leading to genuine NLO soft gluon resummations rely on NNLL accuracy, which is not yet available for g_1^c, see footnote 3.

We plot the ratio $g_1^c\,(\mu)\,/g_1^c\,(\mu = m)$ over a range $m \le \mu \le 10\mathrm{GeV}$ and fix $m = 1.5\,\mathrm{GeV}$, $Q^2 = 10\,\mathrm{GeV}^2$ and $x = 0.05$. In Fig.5a we use the parametrization GS A of Ref.[36], but the following conclusions hold also for the gluon density in the valence scenario of Ref.[37]. We

find at NLO that the corrections based on LL accuracy only are not sufficient to reduce the scale dependence. Clearly, soft gluon effects need to be approximated at least to NLL accuracy to achieve the desired result, a feature that has also been noticed in studies of unpolarized heavy quark production [25]. Our best estimate for g_1^c makes use of the exact expression for the coefficient function $\Delta c^{(1,1)}$ of Eq.(24). However, since it differs only slightly from the NLO result to NLL accuracy for the chosen parameters in Fig.5, we do not display it here.

In Fig.5b we repeat the analysis for the parametrization GS C of Ref.[36]. In this case, the NLO and NNLO approximations do not reduce the scale dependence. This is due to the oscillating gluon density, which leads to g_1^c being close to zero at the x-value chosen, and even causes g_1^c to change sign, depending on the scale.

To summarize, the study of the charm structure function as in Figs.4 and 5 shows that the soft gluon estimates of higher order corrections to the coefficient functions are well under control and give stable predictions for g_1^c at scales Q^2 not too large and $x \gtrsim 0.01$. On the other hand, in the chosen kinematical range the effects of higher orders do not upset the sensitivity to the polarized gluon distribution function, with different gluon parametrizations leading to qualitatively different behaviour for g_1^c. Therefore, measurements of the charm structure function allow to further constrain the polarized gluon density. Finally, we found that the sizable variation of g_1^c at leading order due to different values of the charm mass are not reduced by our approximate higher order corrections.

4 Conclusions

In this letter we have investigated the effects of soft gluons in the case of polarized lepto-production of heavy quarks. Our study helps to keep theoretical control to next-to-leading logarithmic accuracy over large higher order corrections due to threshold logarithms and we have presented analytical results for the all-order resummed cross section in single-particle inclusive kinematics.

Moreover, we have provided both analytical and numerical results for the approximate NLO and NNLO perturbative corrections to the deep-inelastic charm structure function g_1^c, to NLL accuracy. As a consequence of the scattering reaction being largely driven by initial state gluons with not much more energy than required to produce the final state, we have found our analysis of g_1^c to be well applicable in the kinematical range accessible to the HERMES and COMPASS experiments. In this region, our NLO and NNLO estimates can help to reduce theoretical uncertainties and may assist in the theoretical interpretation of future g_1^c measurements.

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